DOCUMENT RESUME

ED 162 899 SE 025 428

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TITLE Mathematical Problem Spaces of the Mathematically Anxious and the Mathematically Comfortable. Paper Presented at a Meeting of the Western Psychological Association, San Francisco, California, April 1978.
SPONS AGENCY California State Univ., Fullerton.
PUB DATE 78
NOTE 23p.; Not available in hard copy due to marginal legibility of original document

EDRS PRICE MF-$0.83 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Algebra; *Anxiety; *College Mathematics; *Concept Formation; *Educational Research; Error Patterns; Higher Education; Mathematics Education; *Problem Solving; *Research
IDENTIFIERS Research Reports

ABSTRACT

Verbal protocols were collected from math-anxious and math-comfortable college students while they solved algebra problems. These protocols were then examined for differences in problem-solving processes. Differences occurred in the use of two basic strategies: generating values and symbolic transformations. Math-anxious students generated more values; math-comfortable students made more symbolic transformations. The data suggest that for the math-anxious students the real problem is not numerical; rather, it is the inability to apply symbolic procedures. (MP)

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# Mathematical Problem Spaces of the Mathematically Anxious and the Mathematically Comfortable

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Running Head: Problem Spaces

SE 025 428

## Abstract

Verbal protocols were collected from math anxious and math comfortable college students while they solved algebra problems. Methods for examining these protocols for differences in problem solving processes have been developed by Newell and Simon (1972). Wallas' (1926) problem solving phases (preparation, incubation, illumination, and verification) were used to develop a problem solving model which included operators used by both groups. This aided in determining the point in the problem solving process at which the math anxious encounter difficulty. Differences occurred during the incubation phase where two basic strategies were involved: generate values and symbolic transformations. Math anxious generated more values; math comfortable made more symbolic transformations. There were no differences between the two groups for arithmetic errors but the math anxious made significantly more conceptual errors. The data suggest that for the math anxious the real problem is not numerical, rather it is the inability to apply symbolic procedures. When applied, they are inappropriate and conceptual errors are generated.

# Mathematical Problem Spaces of the Mathematically Anxious and the Mathematically Comfortable[1]

This study explores differences in mathematical problem solving processes between math anxious and math comfortable college students. The following describe some traits characteristic of the highly math anxious individual:

(1) They not only express panic but also an overwhelming belief that they cannot solve any kind of problem entailing algebra.

(2) They express the belief that the problem they are trying to solve has a simple solution, but that they are too dumb to see it.

(3) They frequently express frustration because they believe they "should know how to solve the problem, after all they did take at least one course in algebra."

(4) Some even express an aversion as well as apprehension to entering the building where mathematics is taught.

What deficits in mathematical problem solving are associated with these high levels of math anxiety?

Problem solving processes are accessible by analyzing what a person is thinking while solving a problem. Research by Newell and Simon (1972) has provided us with a means for accomplishing this through verbal protocols. By having a record of an individual's utterances while problem solving, many differences in processes may be examined.

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1. This undergraduate research project was supported in part by a student grant from the Departmental Association Council of California State University, Fullerton. This paper was presented at a meeting of the Western Psychological Association, San Francisco, April, 1978.

What typifies thinking while an individual is solving a problem? An important theoretical distinction has been expressed by Wertheimer (1959) who dichotomizes thinking into reproductive and productive classifications. Reproductive thinking consists of accessing and applying a previously used solution procedure (emphasized in the associationistic view). Productive thinking, on the other hand, involves a novel, creative, insightful construction of a solution. Newell and Simon (1972) conceptualize an individual as constructing a problem space which directs the individual towards a solution. They define a problem space as consisting of (1) an initial knowledge state for a problem including the given information and the goal, (2) a set of operators and rules for applying them, and (3) the total, potential knowledge available to a problem solver when a sequence of operators are applied. Reproductive thinking is incorporated in their system whenever a previously acquired solution strategy (PASS) is available. However, all problems do not involve application of PASS's. We need to understand how the problem solver attains a solution from the constructed problem space. The authors believe the representation of a problem space or the task environment is critical in determining the composition of the problem space.

That there is both a cost and a benefit in applying a PASS is indicated by Reed's (1977) work. He used problem analogies (problems with similar task environments) to determine if transfer between two similar problems would be

facilitated. He presented subjects with two problems for which the problem spaces were identical except that one of the problems had an additional constraint. He asked subjects to solve two river crossing problems: the "Jealous-Husbands" and the "Missionary-Cannibal" problems. The similarities can be seen if the husbands are substituted for missionaries and wives replace the cannibals. The only distinction between the two problems is that the husbands and wives must be paired. The results he obtained were not altogether intuitive. He found that for an analogy to be effective, the subject must be aware that an analogy between the problems exists. They need to remember not only how they solved the analogous problem but also how to translate the old solution into usable operators for the new problem. The translation must also define a unique relationship between operators for the two problems. When subjects were not informed of the relationship between the two problems, there was no transfer. When the subjects were informed, transfer occurred but only in one direction. Solving the "Jealous Husband" facilitated solving of the "Missionary-Cannibal" problem, but the problems presented in the reverse order did not have the same results. There are reasons for the asymmetry which need not be discussed here, the main point about the study is that PASS's are not always applied.

The individual needs to assess the costs and benefits of applying a PASS. The cost of a PASS is that during translation, problem solvers have to map a correspondence. This may be costly or time consuming. The benefit, of course, is that a

solution is sometimes at hand. Thus, there may be differences in the degree to which PASS's are applied. The math anxious may either have difficulties in doing so or they have an unwillingness to apply a PASS.

The present study compare's differences between math anxious and math comfortable subjects in algebraic problem solving. In order to answer this question, it is useful to first provide some descriptions of how subjects construct problem spaces and search through them for algebraic problems. A hypothetical problem-solving analysis is shown in Figure 1.

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Figure 1 about here.
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This analysis is shaped by Wallas's (1926) distinction between the four phases in problem solving: preparation, incubation, insight, and verification. Secondly, it is important to specify what relationship exists between any operators the subjects use and the flow chart shown in Figure 1.

From previous work with protocols, we can divide operators into the following six categories:

(1) Examine the Problem (EP). This includes rereading the entire problem or any portion thereof.

(2) Generate Inferences (GI). This includes any conclusion drawn from information stated in the problem.

(3) Generate Values (GV). Specific values are substituted for the variables in the problem. This includes trying to solve by trial and error.

(4) Symbolic Transformations (ST). This involves representing verbal descriptions with symbols as well as substituting a set of symbols for one symbol.

(5) Evaluate an Operator (EO). This involves evaluation of the outcome of an operator regarding its accuracy, inaccuracy, or uncertainty about its usefulness.

(6) Expression of Emotion and Uncertainty (EEU). This involves remarks of anxiety, anger, nervous humour, and general uncertainty (totally lost regarding the next step). Some examples are: "I never was good at word problems", "This is a dumb problem, can we go on to the next one?", "Why don't you do a survey on this, then we don't have to bother about this one", "I'm just plain stumped".

Conceptual and arithmetic errors may also be analyzed. Conceptual errors are errors made with reference to conceptual aspects of the specific problem, whereas arithmetic errors are those made in basic arithmetic operations.

The above operators are believed to fit into the problem solving model of Figure 1 as follows:

(1) The preparation phase includes operators EP and GI.

(2) A search for the appropriate PASS occurs here.

(3) A PASS is recognized as being useful.

(4) A chosen PASS is applied, resulting in ST operators.

(5) If a PASS is successful, insight occurs (stage 9). Success includes evaluating an operator as accurate. If a PASS is unsuccessful, the individual returns to search for another PASS (stage 2). The individual may have evaluated

an operator (EO) as inaccurate.

(6) If no PASS is found, a trial and error strategy is pursued which involves the operator GV.

(7) If the trial and error is successful, a kind of insight occurs (stage 9) and it is apparent that a solution has been found. An EO may occur here.

(8) If repeatedly unsuccessful, either frustration occurs and emotional comments are made, or the individual becomes lost in the problem and expressions of emotion and uncertainty are made.

(9) Insight occurs here and it is recognized that a solution has been found.

(10) The individual may return to check the accuracy of a solution: several types of operators may occur here.

By observing protocols and seeing how the frequency of operators are distributed into classes, we may test a variety of hypotheses.

What hypotheses may be offered to account for the differences between the math anxious and math comfortable and how might these be tested? Several hypotheses are possible. Perhaps the math anxious have difficulty in getting an initial description of the problem. They may allocate an insufficient amount of resources to a description of the problem. The opposite can also occur; it could be that a fixation occurs at this stage of problem solving resulting in excessive resources being allocated to this stage. These alternatives can be tested by looking at the proportion of operators used during the preparation phase. That is, if the math anxious allocate less operators

than do the math comfortable to the preparation phase (EP and GI operators) in proportion to those used in the preparation and incubation stages combined, then the math anxious can be said to be blocked at the preparation stage. For this information, it is necessary to look at the proportion of operators allocated to examine the problem and generate inferences compared to all operators combined:

$$\frac{EP + GI}{EP + GI + GV + ST + EO + EEU} \qquad (1)$$

If they allocate more operators to this stage than do the math comfortable, then they may be fixating at this stage.

As a second hypothesis, perhaps the problem for the math anxious is in identifying and applying a PASS. If this is true, relatively many trial and error attempts would occur. Since applying a PASS and making trial and error attempts are the two basic strategies, one of the two would be preferred. If the math anxious allocate more operators to trial and error attempts (GV operators) than to symbolic transformations (ST operators), then

$$\frac{GV}{GV + ST} \qquad (2)$$

would be greater for the math anxious subjects.

For a third hypothesis, perhaps the math anxious worry more about the consequences of operators. If so, their concern would be reflected by the significantly larger proportion of operators allocated to evaluation of operators (EO) and expressions of emotion and uncertainty (EEU) than do the math comfortable. This would be seen by a larger proportion of those two operators when compared to all operators combined:

$$\frac{EO + EEU}{EP + GI + GV + ST + EO + EEU} \cdot \qquad (3)$$

## Methods

Subjects. Subjects consisted of 22 male and female introductory psychology students who had declared themselves to be math anxious or math comfortable. They received class credit for their participation. Subjects ranged in age from 17 to 24 years and were required to have had beginning algebra sometime within the preceeding five years, however, they were not disqualified if their math education exceeded beginning algebra. Subjects were assigned to either the math comfortable or math anxious group according to where on a 1 to 10 scale they reported themselves regarding their levels of math anxiety. Those in the 1 to 5 range of the scale were assigned to the math comfortable group and those in the 5 to 10 range of the scale were assigned to the math anxious group Two subjects who fell in the middle of the scale (5) were eliminated. Two others were eliminated due to faulty recording equipment.

Material. The MARS (Suinn,1972) is a 98-item questionnaire in which each item is to be answered on a five point scale from 1="not at all" to 5="very much" depending on how much anxiety is aroused by the situation described. The items describe different situations both in ordinary life and in an academic setting involving number manipulations and the use of mathematical concepts. Some examples are: "Being asked to add up 976 + 777 in your head", "Doing a word problem in algebra", "Meeting

your math teacher while walking on campus", "Being told how to interpret probability statements".

Three subtests from the Differential Aptitude Test (DAT) were administered: Numerical Abilities, Verbal Reasoning, and Abstract Reasoning. Due to time constraints, only the even numbered items from each subtest were presented.

Three algebra problems were selected. Two problems involved simultaneous linear equations: one was in mathematical notation and one was in words. The third was a word problem for which there was no familiar solution procedure. The problems are listed below.

(1) Solve for x $\quad 3x + 2y = 2$
$\quad\quad\quad\quad\quad\quad\quad\quad 5x + 6y = 4$

(2) At one basketball game 1845 tickets were sold. Students paid two dollars each for their tickets, and nonstudents paid five dollars each. If the total receipts for the game were $4500, how many tickets were sold?

(3) Two math professors meet on the street, they have not seen each other for several years. "Hi, how are you?" "Fine, how are you?". "How many kids do you have now?". "Three." "How old are they?" "The product of their ages is 36." "I still don't know old they are." "The sum of their ages is the address of that place we used to play chess." "I still don't know how old they are." "The oldest one looks like me." "Now I know how old they are." What are the ages of the children?

Design. A 2(comfortable vs. anxious) x 9(subjects) x 3(problem) mixed design was used. Anxiety level was a measure used between subjects. The number of problems presented was a repeated measure.

Procedure. The subjects were first asked to assess their anxiety towards mathematics on a scale from 1 to 10 where 1 represented "very comfortable with math" and 10 indicated "extreme discomfort with math". The instructions to the subjects were then read, they are as follows:

> This experiment is concerned primarily with what people are thinking as they are in the process of solving a mathematical problem. This is done by having you think out loud. Some people approach math with almost no discomfort, whereas some people approach math with a great deal of discomfort, or math anxiety. This experiment is concerned mainly with what people are thinking as they are in the process of solving a problem. This experiment will be looking as the differences in mathematical problem processes between two groups: those with high levels of math anxiety and those with low levels. This can be done only by your thinking out loud. Math anxiety is a somewhat common problem that varies only in degree according to the situation. In this experiment, I am concerned mainly with what people are thinking as they are in the process of solving a problem.
> This experiment focuses on one kind of math problem, algebra problems. I will present you with 3 algebra problems and I need to have you think out loud while you are solving them. I recognize this will feel awkward, but it is essential if I am to gather the information I need. I will be tape recording all your comments. Again, as clearly and loudly as you are able, please give voice to any thoughts or feelings you may have as you are presented with the problems although it may feel unusual. Your anonymity will be respected. You have the right to leave if you feel a great amount of discomfort but I hope you will stay as math anxiety is a pervasive problem.

The subject was asked what their understanding of the instructions were and, if necessary, the appropriate portions were reread.

Each subject was then presented with a randomized order of the three problems. Ten minutes were allowed for each problem. A microphone was placed as close as possible to the subject. (Subjects had a tendency to speak indistinctly.) No questions

were answered within these 10 minutes except to reassure the subject that the problem they were working on had a solution. If after the first 10 minutes the problem remain unsolved, the subject was offered three problem facilitators. They were: (1) In this problem, what do you see as given information and constraints? (2) What kind of relationships do you see in this problem? (3) What kind of information could have been included in this problem that would have helped you? The solutions were shown to each subject following each problem.

Following the three problems, the three DAT subtests were administered for 7 minutes each. Finally, the MARS (Suinn,1972) was administered. The subject was directed to complete the entire questionnaire without regard to time. The debriefing took as much time as necessary to assure the subject that performance was not a reflection of their intellect but that the primary focus of interest in this experiment was the problem solving processes.

## Results

A positive relationship was found for the MARS (Suinn,1972) and self report scores, r=.88. High scores on both scales represented very similar anxiety levels. The mean MARS score for this study was 224.2 with a standard deviation of 66.8. This is slightly higher than that obtained from the normative data for which the mean score was 215.8 with a standard deviation of 65.29.

The separate categories of operators were compared for differences between the two groups. Table 1 lists the mean proportion of operators for each category.

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Table 1 about here.

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There were no differences between the two groups in examining the problem (EP), in generating inferences (GI), and in the number of arithmetic errors (AE) made. The math anxious allocated significantly more operators to evaluating operators (EO), expressions of emotion and uncertainty (EEU), and made significantly more conceptual errors (CE).

Table 2 lists the proportion of operators allocated at stages of the problem solving model of Figure 1 for problems

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Table 2 about here.

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1 and 2 combined and for problem 3. Formula 1 tested the hypothesis that the math anxious had difficulty in describing the problem. The results indicate that there were no differences between the two groups in describing the problem for all 3 of the problems but more operators were used in problem 3.

Formula 2 tested the hypothesis that the math anxious had difficulty in applying a PASS. The math anxious were found to generate significantly more values than did the math comfortable in problems 1 and 2. They were unable to apply PASS's. No differences were found between the two groups for problem 3.

Formula 3 tested the hypothesis that the math anxious worried more over the consequences of their operators. The results indicate that the math anxious did worry more for they generated significantly more operators at this point than did the math comfortable for problems 1 and 2 but not for 3. There were no differences between the two groups for problem 3.

The two groups were compared for their scores on the three DAT subtests: Numerical Ability, Verbal Reasoning, and Abstract Reasoning. The results are listed in Table 3.

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Table 3 about here.

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There were no differences found between the two groups in their scores for verbal reasoning and abstract reasoning. The math comfortable, however, scored significantly higher in numerical abilities.

## Discussion

Differences in mathematical problem solving strategies were found to vary according to the level of math anxiety. The math comfortable focused more on processes whereas the math anxious concentrated more on consequences. The real problem for the math anxious is not numerical; it is the inability to apply symbolic procedures. When these procedures are applied, they are inappropriate and conceptual errors are generated.

Differences in problem type affected the proportion of operators allocated to the various stages of problem solving

as can be seen in table 2. Problems 1 and 2 were solvable by conventional procedures involving the use of PASS's whereas problem 3 required a nonconventional solution entailing mathematical logic. Although both groups performed similarly in describing the problem within each comparison, they both required more operators for the problem with the nonconventional solution.

In identifying and applying PASS's there were wide differences between the two groups for problems 1 and 2. These problems were solvable with PASS's. This was not the case, however, in problem 3. Here PASS's were inappropriate and the math comfortable, like the math anxious, pursued the other strategy, generating values. This problem, then, has the effect of narrowing the differences between the two groups.

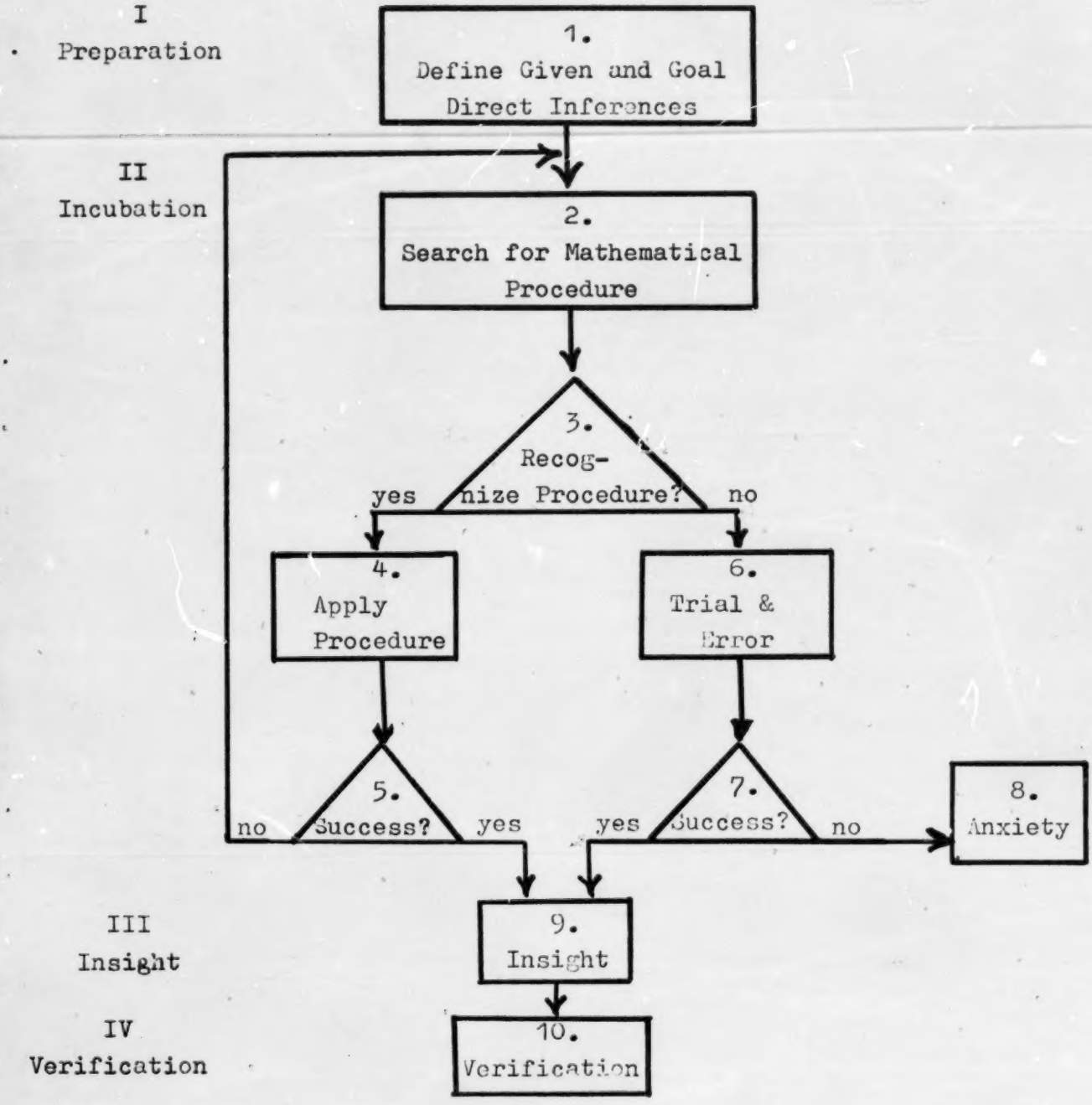
Similar concern over the consequences of operators was expressed by both groups in problem 3. This again was probably due to the lack of any appropriate PASS's. When compared with the significantly smaller concern by the math comfortable for consequences in problems 1 and 2, this seems to be the case: PASS's make a difference

The MARS (Suinn,1972) and self report scales were found to indicate a similar level of math anxiety as evidenced by the high correlation between the two scales. This suggests that the two methods of assessing math anxiety were relatively interchangeable.

Results from the DAT subtests (see Table 3) suggest both groups have similar abilities in the areas of verbal and abstract reasoning. This is not the case for numerical abilities; the math

anxious perform significantly poorer as a group. Perhaps the difference was due to fatigue; the subtests were not presented until after all the problems had been attempted. Also the numerical abilities subtest is not without some symbolic procedures (e.g. proportions). This would tend to further support the finding that the real problem for the math anxious is an inability to use symbolic procedures.

A problem in this study was a lack of control for mathematical abilities. Perhaps this difficulty could be overcome partially by testing children who have had less experience with mathematics. Further work in this area is necessary to understand the causes of math anxiety in order to prevent its development in young children.




Problem Solving Model

Figure 1

Mean Proportion of Operators
For Each Category

| | Anxious | Comfortable | F |
|---|---|---|---|
| Examine Problem (EP) | 24.5 | 28.4 | .75 |
| Generate Inferences (GI) | 26.5 | 25.0 | .003 |
| Generate Values (GV) | 13.0· | 5.3 | 5.12* |
| Symbolic Transformations (ST) | 14.9 | 30.2· | 11.17* |
| Evaluate Operators (EO) | 11.6· | 6.6 | 14.66* |
| Expressions of Emotion and Uncertainty (EEU) | 9.7· | 2.1 | 11.11* |
| Conceptual Errors (CE) | 9.1· | 2.7 | 12.76* |
| Arithmetic Errors (AE) | 3.8 | 2.6 | .89 |

*$p < .05$

Table 1.

Proportion of Operators Allocated at Stages of the Problem Solving Model

| | Problems 1 and 2 (conventional solution) | | | Problem 3 (nonconventional solution) | | |
|---|---|---|---|---|---|---|
| | MC | MA | t | MC | MA | t |
| Describe the Problem (EP + GI) | .48 | .45 | .5 | .68 | .62 | 1.00 |
| Identify and Apply a PASS (GV) | .08 | .36 | 3.29* | .52 | .66 | .82 |
| Consequences (EO + EEU) | .07 | .22 | 3.75* | .14 | .19 | 1.25 |

* $p < .05$

Table 2.

Mean Scores for the DAT Subtests

| | Math Anxious | Math Comfortable | F |
|---|---|---|---|
| Numerical Abilities | 7.8 | 11.8 | 5.62* |
| Verbal Reasoning | 17.1 | 16.1 | .22 |
| Abstract Reasoning | 17.7 | 20.2 | 2.16 |

* $p < .05$

Table 3.

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